COST Action CaLISTA
CA21109

Department of Mathematics
Faculty of Science
University of Zagreb

Quantum Groups and Lie Groups

Book of abstracts

WG3 meeting

September 25 – 26, 2024
Zagreb, Croatia

COST action CaLISTA

Symmetry is a central unifying theme in mathematics and physics. This Action focusses on symmetries realized through Lie groups and Lie algebras. In addition to the spectacular achievements in representation theory, and differential geometry, Lie theory is also exceptionally important for the formalization of fundamental physical theories. CaLISTA aims to advance cutting-edge research in mathematics and physics through a systematic application of the ideas and philosophy of Cartan geometry, a thoroughly Lie theoretic approach to differential geometry. In addition to making major progress in Cartan geometry itself, CaLISTA aims to develop crucial applications to integrable systems and supersymmetric gauge theories. Quantum groups and their quantum homogeneous spaces come into the play as a bridge between these topics: quantum groups stem originally from the R-matrix formulation in integrable systems, and their homogeneous spaces offer prototypical examples of noncommutative parabolic geometries. Parabolic geometry is the first and possibly the most important example of Cartan geometry, and one of the main aims of CaLISTA is to obtain a quantum generalization.

Surprisingly, Lie theory and Cartan geometry play a role in an exciting new interpretation of the differential structure, and related dynamics, of models for popular algorithms of vision like Deep Learning and the more recent Geometric Deep Learning. CaLISTA aims to investigate and improve on these techniques. CaLISTA will provide essential mathematical models with far-reaching applications, placing Europe among the leading actors in these innovative research areas.

Action keywords: Lie Theory - Cartan Geometry - Quantum Groups - Integrable Systems - Vision

How can I participate?

- Read the Action Description
- Inform the Main Proposer/Chair of your interest (email)
- Apply to join your Working Groups of interest
- Please note, Management Committee nominations are carried out through the COST National Coordinators

Action Details.

- Start date - 14/10/2022
- End date - 13/10/2026
- Website: `https://site.unibo.it/calista/en`

Working Groups.

WG1 Cartan Geometry and Representation theory, leader Prof. Andreas Čap
WG2 Integrable Systems and Supersymmetry, leader Prof. Simonetta Abenda
WG3 Noncommutative Geometry and Quantum Homogeneous Spaces, leader Dr. Reamonn Ó Buachalla
WG4 Vision models, leader Prof. Jesus Angulo
WG5 Dissemination and Public Engagement, leader Dr. Juraj Tekel

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1. Schedule

	25.09	26.09
09:00 - 09:30	Registration	
09:30 - 10:20	Balagović	Fioresi
10:20 - 10:40	*Coffee break*	
10:40 - 11:30	Li	Krähmer
11:35 - 12:25	Somberg	Škoda
12:25 - 15:00	*Lunch break*	
15:00 - 15:50	Part	Ó Buachalla
15:50 - 16:10	*Coffee break*	
16:10 - 17:00	Prlić	Carotenuto
17:05 - 17:25		Novak

2. Abstracts

Alessandro Carotenuto (*University of Bologna*)

Title: **Convex orderings and quantum tangent spaces**

Abstract: One of the most remarkable phenomena that occurs when we take in consideration a quantum enveloping algebra $U_q(\mathfrak{g})$ is the fact that a PBW basis is not uniquely defined but rather depends upon the choice of a reduced decomposition of the longest element w_0 in the corresponding Weyl group. For each of such choices we have a set of so-called quantum root vectors as defined by Lusztig, together with a convex order defined on the positive roots of $\mathfrak{g}$. In this work in collaboration with P. Papi, we study how the combinatorics of a reduced decomposition of w_0 is connected to the existence of a quantum tangent space for the full quantum flag manifold of $\mathfrak{g}$.

Martina Balagović (*Newcastle University*)

Title: **Towards bases for representations of QSP coideal subalgebras**

Abstract: I will discuss an ongoing project on representations of certain quantum symmetric pair coideal subalgebras of quantum groups. By recent work of Stefan Kolb and Jake Stephens, such algebras have equivalents of roots, satisfy the PBW theorem, and their irreducible finite dimensional representations have weights and can be classified in terms of weights. We would now like to construct bases for these representations compatible with their relationship with quantum groups.

I will explain what are these algebras, why they are of interest, how they relate to quantum groups, and what Kolb and Stephens can show about their representations. I will then describe desired properties of the bases of these representations that one could hope for in analogy with classical Lie theory, list questions about structures which control these bases, and partially answer some of these questions. Joint work in progress with Stefan Kolb.

Rita Fioresi (*University of Bologna*)

Title: **Reduction of Quantum Principal Bundles**

Abstract: We develop the theory of reduction of quantum principal bundles over projective bases. We show how the sheaf theoretic approach can be effectively applied to certain relevant examples as the Klein model for the projective spaces; in particular we study in the algebraic setting.

Ulrich Krähmer (*Dresden University of Technology*)

Title: **The ring of differential operators on a monomial curve is a Hopf algebroid**

Abstract: The ring of differential operators on a cuspidal curve whose coordinate ring is a numerical semigroup algebra is shown to be cocommutative and cocomplete left Hopf algebroid, which essentially means that the category of D-modules is closed monoidal. If the semigroup is symmetric so that the curve is Gorenstein, it is a full Hopf algebroid (admits an antipode), which means that the subcategory of those D-modules that are finite rank vector bundles over the curve is rigid. Based on joint work with Myriam Mahaman.

Jianrong Li (*University of Vienna*)

Title: **Newton polytopes, quantum affine algebras, and scattering amplitudes**

Abstract: In this talk, I will talk about a connection between Newton polytopes, representations of quantum affine algebras, and scattering amplitudes in physics. We give a systematic construction of prime modules (including prime non-real modules) of quantum affine algebras using Newton polytopes. We give a general formula of u-variables which appears in the study of scattering amplitudes in physics using prime tableaux (corresponding to prime modules of quantum affine algebras of type A) and Auslander-Reiten quivers of Grassmannian cluster categories. This is joint work with Nick Early.

Josip Novak (*University of Zagreb*)

Title: **Reflective localizations of bicategories**

Abstract: In this talk, we give an overview of the theory of localization of bicategories via bicalculus of fractions. The bicategory of fractions with respect to a class of morphisms was introduced by Pronk in 1996 as a generalization of the Gabriel-Zisman theory of localization. The main goal was to show that étendues and stacks arise as bicategories of fractions of appropriate categories of groupoids, e.g. orbifolds as localizations of Lie groupoids. We expand on the theory of localization of bicategories by giving a characterization of reflective localizations.

Réamonn Ó Buachalla (*Charles University in Prague*)

Title: **Noncommutative geometry of the quantum flag manifolds: the rank 2 case**

Abstract: The quantum flag manifolds are a fascinating class of quantum homogeneous spaces that q-deform the function algebra of the complex flag manifolds. The quantum flag manifolds of irreducible type form one of the best understood classes of noncommutative geometries in the whole theory of quantum groups. This is due in large part to the discovery by Heckenberger and Kolb in the early 2000s that the classical de Rham complex of the irreducible flags admits an essentially unique q-deformation. The question of how to extend beyond the irreducible setting has been the subject of much interest recently. In this talk I will present the state of the art for those quantum flag manifolds associated with simple Lie algebras of rank 2. Moreover, time permitting, I will briefly discuss the rank 3 and 4 cases.

Fedor Part (*Institute of Mathematics, Czech Academy of Sciences*)

Title: **On existence of coideal deformations of the Kronecker embedding**

Abstract: The Kronecker embedding of $\mathfrak{gl}(n)\oplus\mathfrak{gl}(m)$ into $\mathfrak{gl}(nm)$ is defined as differential of the tensor product embedding of $GL(n)xGL(m)$ into $GL(nm)$. We investigate the question of whether it is possible to deform $U(\mathfrak{gl}(n)\oplus\mathfrak{gl}(m))$ into a quantum subgroup of $U_q(\mathfrak{gl}(nm))$ with certain structural properties. We draw upon the theory of ι-quantum groups represented by quantum symmetric spaces and primarily look for deformations that are coideal and/or have (quasi-)K-matrix, PBW bases, canonical and crystal bases. The main motivation is a construction of suitable crystal bases for the Kronecker quantum subgroup with application to branching problem for the Kronecker embedding.

Ana Prlić (*University of Zagreb*)

Title: **Dirac operators in representation theory**

Abstract: This talk gives an overview of the use of the Dirac operators in representation theory. Dirac operators were introduced into representation theory by Parthasarathy as a tool for constructing discrete series representations. D. Vogan studied an algebraic version of the Dirac operator and in 1997 conjectured that the Dirac cohomology of an admissible $(\mathfrak{g}, K)$-module, if nonzero, determines its infinitesimal character. Vogan's conjecture was proved by Huang and Pandžić in 2002.

In this talk, we will explain the advantages of the use of the Dirac operators in representation theory, and give an overview of some known results with an emphasis on a recent paper by Pandžić, Prlić, Souček, and Tuček which provides an alternative proof of the classification of unitary highest weight modules originally proved by Enright, Howe and Wallach and independently by Jakobsen.

Zoran Škoda (*University of Zadar*)

Title: **Global Gauss decomposition of quantum groups**

Abstract: Gauss elimination procedure has been extended to linear systems over noncommutative rings. For example, Oystein Ore considered the case of Ore domains in 1931. In a modern expression of the Gauss procedure, Gelfand and Retakh decomposed the generic square matrices with noncommutative entries into the products of lower triangular unidiagonal, diagonal and upper triangular unidiagonal matrices in terms of quasideterminants which are certain well-behaved noncommutative rational functions attached to matrices with noncommutative entries. Quasideterminants involve inverses, hence the decomposition holds only after localizing the algebra. If the matrix is multiplied by a permutation matrix, the new matrix can be decomposed in another localization and, informally speaking, all permutation matrices together determine a global cover of the noncommutative spectrum of the algebra generated by entries of generic matrices. In FRT approach, coordinate rings of quantum groups are given in terms of matrices of generators. For quantum linear groups, the cover by localizations stemming from the noncommutative Gauss decomposition can be fully formalized and the formulas for the entries of the lower triangular matrices in the decomposition extend to algebra maps from lower quantum Borel subgroup to the localizations. Moreover these algebra maps are compatible with induced comodule structure over Borel and become cleavages for Hopf algebraic smash products or, in a geometrical language, local sections of a locally trivial principal bundle over the quantum flag variety. The result appropriately extends to parabolic subgroups inducing a fine tower of bundles among partial quantum flag varieties, and conjecturally it extends to other quantum groups of FRT type. The fully noncommutative case is related to the universal noncommutative flag varieties where the exactness properties of involved localizations hold only for finitely generated projectives, beyond which one needs higher categorical treatment involving Bousfield localization and derived descent.

Petr Somberg (*Charles University in Prague*)

Title: **Realization of generalized Verma modules for quantum groups**

Abstract: We introduce suitable technical environment, allowing a (rather concrete and explicit) realization of generalized Verma modules for quantum groups. This is a first step in the analysis of induced representations for quantum groups.

3. Useful information

The conference venue is the Department of Mathematics, Faculty of Science, University of Zagreb (Bijenička street, 30). The lectures will take place in the lecture hall A001 located on the ground floor. The coffee breaks will take place in the lecture hall A002.

3.1. Bus schedule from Kaptol to the Department of Mathematics (Bijenička). The bus ride from the bus station *Kaptol* (which is located near the cathedral) is less then 10 minutes. Get off the bus stop *Bijenička* (the third stop, note that all stops are on request) which is located after a roundabout. You can take one of the following buses: 106, 201, 203, 226, 238 (every bus from Kaptol is fine *except* 105).
Bus 106: 6:20 6:35 6:50 7:10 7:30 7:50 8:10 8:30 8:50 9:10 9:30 9:50 10:10 10:30 10:50 11:10 11:30 11:50 12:10 12:30 12:50 13:10 13:30 13:50 14:10 14:30 14:50 15:10 15:30 15:50 16:10 16:30 16:50 17:10 17:30 17:50 18:10 18:30 18:50 19:10
Bus 201: 4:40 5:05 5:30 5:55 6:20 6:45 7:10 7:35 8:00 8:25 8:50 9:15 9:40 10:05 10:30 10:55 11:20 11:45 12:10 12:35 13:00 13:25 13:50 14:15 14:40 15:05 15:30 15:55 16:20 16:45 17:10 17:35 18:00 18:35 19:10 19:45 20:20 20:55 21:30 22:05 22:40 23:15
Bus 203: 4:45 5:10 5:35 6:00 6:25 6:50 7:15 7:35 8:05 8:30 9:00 9:20 9:45 10:15 10:35 11:00 11:30 11:50 12:15 12:40 13:05 13:35 13:55 14:20 14:45 15:10 15:35 15:55 16:25 16:50 17:15 17:40 18:00 18:30 19:00 19:20 19:45 20:10 20:35 21:00 20:25 20:55 22:15 22:40 23:05 23:35 0:10
Bus 226: 4:30 5:00 5:30 6:05 6:30 7:05 7:50 8:25 9:05 9:35 10:05 10:45 11:20 11:55 12:25 13:05 13:40 14:15 14:45 15:25 16:00 16:35 17:05 17:40 18:20 18:55 19:30 20:05 20:40 21:15 21:50 22:25 23:00 23:00 0:01
Bus 238: 6:30 7:15 7:55 9:00 9:50 10:40 11:30 12:20 13:10 14:00 14:50 15:40 16:30 17:20 18:10 19:10

More information is available on the website `www.zet.hr`.

3.2. Conference dinner. The conference dinner will take place at the restaurant "Batak Kvatrić". The restaurant address is str. Jakova Gotovca 1, 10000, Zagreb, tram stop "Kvaternikov trg" (accessible by trams 12, 11 from the main square). The restaurant is located in the yard. The entrance is from Vlaška street or from Petrova street.

Organisation committee:	Karmen Grizelj (University of Zagreb)
	Andrey Krutov (Charles University)
	Pavle Pandžić (University of Zagreb)
	Ana Prlić (University of Zagreb)
Webpage:	`https://sites.google.com/view/glq2024/home`